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A Study on Brexit: Correlations and Tail Events Distribution of Liquidity Measures

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Abstract

Liquidity describes the degree to which an asset or security can be quickly bought or sold in the market without affecting the asset's price. In this study, some of the existing liquidity measures are studied and analyzed during Brexit. We examine Utilities Select Sector SPDR Fund (Exchange-Traded Fund) components in this study. The time period covers June 16, 2016 to June 30, 2016 which includes Brexit event day. We use high-frequency tick level Trade data, Quote data, and Limit Order Book data. We study the sample of Trade and Quote liquidity measures (TAQL) and Limit Order Book liquidity measures (LOBL). Our study shows that the correlations between these two liquidity groups (TAQL & LOBL) have significant relationship with the returns of the underlying ETF components. Furthermore, the analysis shows that low correlation between TAQL and LOBL indicates high probability of large price change. Finally, we study the empirical distributions, which implies that Brexit generated fatter tails on liquidity measures distributions. This indicates that infrequent (low) liquidity condition occurs more frequently during Brexit.

Keywords: Brexit; Liquidity Measures; Correlation; Tail Events

1 Introduction

In financial markets, liquidity is the term used to describe how easy it is to convert assets to cash. This definition is equivalent to considering low transaction cost, short execution time, and small impact on asset's prices. A liquid market might be characterized as a continuous market where traders can buy or sell any amount of stock immediately (Black (1971)).
Theory and empirical evidence indicate that investors require higher returns on assets with low market liquidity to compensate them for the high transaction cost (Amihud (2006)). Liquidity plays an important role in asset pricing and market microstructure.

Brexit referendum took place on June 23, 2016, and it resulted in 51.9% of voters voting in favor of the UK leaving the EU. The effect of Brexit on economy has a significant short-term and long-term consequence, such as the decrease of GBP value and the increase of inflation rate in the UK. Brexit's effects on New Zealand, Australia and Indian Stock Markets are studied by Abraham (2016), in which economic crisis's effects are investigated and there is no significant effect in the stock markets during the crisis period. Schiereck et al. (2016) analyzed Brexit referendum's effects on the stock and CDS market are analyzed and it is found that Brexit has more significant influence on the short-run drop in stock prices than Lehman's bankruptcy. Liquidity risk and its effects on price during Brexit is studied by Mago et al. (2017).

Tick level data (TAQ and LOB data) was obtained from Thomson Reuters Tick History Database (TRTH) to perform the analysis. All components of a specific ETF (Exchange-Traded Fund) are selected as the targets of this research. In general, an ETF tracks a certain market sector and provides reduced exposure to individual constituents. We consider the Utilities Select Sector SPDR Fund for this study given its significant movement during Brexit referendum studied by Suttmeier (2016).

There more than 65 liquidity measures proposed in existing literature by Salighehdar et al. (2017). Most liquidity measures have been applied to low frequency data framework. Nowadays, high-frequency data is available and high-frequency trading accounts for a large portion of the US equity market (Brogaard (2010)).

In this study, we analyze TAQL measures and LOBL measures using high-frequency data. Traditionally, the liquidity measures have been classified as one-dimensional or multi-dimensional based on the specific liquidity features they are measuring. For example, Bacidore (1997) the number of transactions in a certain period of time, by which frequent transactions reflect high liquidity. Underlying net trade volume (buyer-side volume minus seller-side volume) has the ability to predict stock quotes (Chan et al. (2002)). Turnover and market depth focus on the total trading amount and the total volume of bid and ask volume (Brockman et al. (2000)). Bid and ask spread is frequently used as the indicator to reflect liquidity conditions (Chordia et al. (2001) and Grammig et al. (2001)). There are other liquidity measures concentrating on spread studied by (Hamao and Hasbrouck (1995), Levin et al. (1999), Fleming and

Remolona (1999), Hasbrouck and Seppi (2001), Genccay et al. (2001), and Ranaldo (2008)). Various slopes are introduced to present liquidity in existing literatures by Hasbrouck (2001) and Chordia et al. (2001). There are some ratios describe market liquidity, such as Liquidity Ratio 1, which is known as Amivest liquidity ratio (Elyasiani et al. (2000)). Flow Ratio, and Order Ratio are studied in Ranaldo (2000). Another daily ratio of absolute stock return to its dollar volume is defined as an Illiquidity measure which is known as Amihud liquidity measure. It is concluded that expected stock returns are an increasing function of expected illiquidity (Amihud (2002)).

Besides TAQL mentioned above, there are existing literatures that proposed liquidity measures based on limit order book information. Cost of Round Trip measure is presented by Irvine et al. (2000). This measure gives a numeric value to represent liquidity of the entire LOB. Dispersion measure reflects how distributed limit orders are and Modified Dispersion measure accounts for net dispersion which indicates directions (Kang et al. (2013) and Shen et al. (2016)).

The main goal of this paper is to study the dependency between groups of liquidity measures and the impact of extreme liquidity conditions on underlying price behaviors.

Firstly, we study the relation between existing liquidity measures correlations and price behavior. Our findings show that the correlation between TAQL and LOBL has significant change on the event day compared to the rest of the dataset. By defining a correlation index, we find that low correlation index corresponds to high probability of large price change. We are able to have a better comprehension on how liquidity conditions affect the price of assets by studying a large sample.

Secondly, the tail events of liquidity measures are studied. There are existing literature proposed to detect liquidity measures' distributions, but we are curious about how market events' effects on changing the distributions. We calculate kernel density on each liquidity measure of all ETF components. After studying distributions and quantiles, we conclude that liquidity measures distributions have fatter tails on the event day which indicates that illiquid condition happened frequently during the Brexit. By studying tail events, we are able to understand distributions of liquidity measures and predict potential liquidity conditions by given information.

This paper is organized as follows. In section 2, we provide data and liquidity measures mathematical formations we use in this study, and we show how we calculate these measures. In section 3, we present the empirical results from correlation analysis and tail events study. Finally, in section 4, we give the conclusion of the paper.

2 Data and Liquidity Measures

In this section, we illustrate the data and the liquidity measures studied in this paper. First, the original data is discussed. Second, we analyze the liquidity measures selected, and then talk about how we calculate them using the data.

2.1 Data

The Brexit referendum took place on June 23, 2016 in the United Kingdom (UK) and Gibraltar. We select data from June 16, 2016 to June 30, 2016 (11 trading days) to analyze Brexit's effects on liquidity. 28 components of Utilities Select Sector SPDR Fund (XLU) are selected for analysis, and raw tick-level data is downloaded from Thomson Reuters Tick History Database (TRTH).

Although we selected time window carefully, the high frequency data still contains missing values, such missing values make liquidity conditions unmeasurable, and when we have missing values into liquidity calculations, the results will be unavailable. Sometimes no trade occurs in the first a few seconds or even a few minutes at the beginning of each trading day, and this phenomenon causes frequent missing values in the first a few rows of dataset. In this study, the first a few rows are deleted until trading becomes active to make the dataset more consistent.

Basic variables including average trading price, volume-weighted average trading price, first-trade price, last-trade price, number of trades, trading volume, best bid price, best bid size, best ask price, and best ask size are calculated in this step. Similarly, LOB dataset contains variables including ten levels of best bid price, best bid size, best ask price, and best ask size.

2.2 Descriptions of Liquidity Measures

We provide an overview of liquidity measures studied in this work. Two categories (TAQL and LOBL) of liquidity measures used in this study and their mathematical formulations are displayed in Table 1 and Table 2. The meaning of each variable is displayed in Table 3.

Table 1. Trade and Quote Liquidity measures list

TAQL Measures	Formula
Turnover	$V_t = \Sigma_{i=1}^{N_t} p_i \cdot q_i$
Market Depth	$D_t = q_t^A + q_t^B$
Log Depth	$Dlog_t = ln(q_t^A) + ln(q_t^B)$
Dollar Depth	$DS_t = \frac{q_t^A \cdot p_t^A + q_t^B \cdot p_t^B}{2}$
Spread	$Sabs_t = p_t^A - p_t^B$
Relative Spread (mid)	$SrelM_t = \frac{2 \cdot (p_t^A - p_t^B)}{p_t^A + p_t^B}$
Relative Spread (last)	$Srelp_t = \frac{p_t^A - p_t^B}{p_t}$
Relative Spread (log)	$Srellog_t = \ln(p_t^A) - \ln(p_t^B)$
Effective Spread	$Seff_t = \lvert p_t - p_t^M \rvert$
Relative Effective Spread (last)	$Seffrelp_t = \frac{\lvert p_T - p_t^M \rvert}{p_t}$
Relative Effective Spread (mid)	$SeffrelM_t = \frac{\lvert p_t - p_t^M \rvert}{p_t^M}$
Quote Slope	$QS_t = \frac{Sabs_t}{Dlog_t} = \frac{p_t^A - p_t^B}{\ln(q_t^A) - \ln(q_t^B)}$
Log Quote Slope	$LogQS_t = \frac{\ln(p_t^A) - \ln(P_t^B)}{\ln(q_t^A \cdot q_t^B)}$
Adjusted Log Quote Slope	$LogQSadj_t = LogQS_t \cdot (1 + \lvert \ln(\frac{q_t^A}{q_t^B}) \rvert)$
Composite Liquidity	$CL_t = \frac{2 \cdot (p_t^A - p_t^B)}{p_t^M \cdot (q_t^A \cdot p_t^A + q_t^B \cdot p_t^B)}$
Liquidity Ratio 1 (Amivest)	$LR1 = \frac{V_t}{\lvert r_t \rvert} = \frac{\Sigma_{i=1}^N p_i \cdot q_i}{\lvert r_t \rvert}$
Flow Ratio	$FR_t = V_t \cdot N_t = N_t \cdot \Sigma_{i=1}^N p_i \cdot q_i$
Order Ratio	$OR_t = \frac{\lvert q_t^B - q_t^A \rvert}{V_t} = \frac{\lvert q_t^B - q_t^A \rvert}{p_t \cdot q_t}$
Illiquidity (Amihud)	$Illiquidity_t = average(\frac{\lvert r_t \rvert}{Volume_t})$

Table 2. Limit Order Book Liquidity measures list

LOB Measures	Formula
Cost of Round Trip	$CRT(D) = \frac{\Sigma_{k=0}^{\infty} I_k P_k Q_k - \Sigma_{k=0}^{\infty} I_{-k} P_{-k} Q_{-k}}{D}$
Dispersion	$LD_i = \frac{1}{2}\left(\frac{\Sigma_{j=1}^{n} \omega_j^{Buy} Dst_j^{Buy}}{\Sigma_{j=1}^{n} \omega_j^{Buy}} + \frac{\Sigma_{j=1}^{n} \omega_j^{Sell} Dst_j^{Sell}}{\Sigma_{j=1}^{n} \omega_j^{Sell}}\right)$
Modified Dispersion	$DiffLD_i = \frac{1}{2}\left(\frac{\Sigma_{j=1}^{n} \omega_j^{Buy} Dst_j^{Buy}}{\Sigma_{j=1}^{n} \omega_j^{Buy}} - \frac{\Sigma_{j=1}^{n} \omega_j^{Sell} Dst_j^{Sell}}{\Sigma_{j=1}^{n} \omega_j^{Sell}}\right)$

Table 3. Variables list

Variables	Meanings
N	Number of transactions in a unit time
p	Trading price
q	Trading quantity
p^A	Ask price
q^A	Ask size
p^B	Bid price
q^B	Bid size
p^M	Mid-price between bid price and ask price
r	Underlying return
I_k	Indicator function corresponding to dollar amount
P_k	Bid price (k < 0), ask price (k > 0)
Q_k	Bid size (k < 0), ask size (k > 0)
D	Dollar amount corresponding to daily trading volume
ω	Weight defined as the normalized size of limit orders
Dst_j	Price intervals between j[th] and the next best quote

In Table 1, Transactions Number is the time-related liquidity measure, while Trading Volume, Turnover, Market Depth, and Dollar Depth are volume-related liquidity measures. Bid-ask spread is a widely used liquidity measure, and there are also spread-related liquidity measures listed in the table. Log Spread, Relative Spread, Log Relative Spread with Log Prices, Effective Spread, and Relative Effective Spread are spread-related measures we analyzed in this study. All these measures mentioned above are one-dimensional liquidity measures each considers only one aspect including time, volume, or spread (Von Wyss (2004)). The rest of liquidity measures in Table 1 are multi-dimensional measures constructed by one-dimensional measures. Quote Slope is spread over dollar depth indicating the slope of the line between the bid quote and the ask quote. Log Quote Slope is established to improve distributional properties, and Adjusted Log Quote Slope is to correct the Log Quote Slope for a market moving in one direction (Hasbrouck and Seppi (2001), and Chordia et al. (2001)). Composite Liquidity improve the evaluating capability when spread is not affected. Liquidity Ratio 1 reflects the level of price movement that can be absorbed (Elyasiani et al. (2000)), while Flow Ratio measures whether trading takes place in a few but large transactions or in lots of small trades, and Order Ratio shows market imbalance to turnover (Ranaldo (2000)). Illiquidity is a well-known measure that considers the amount of trading can be absorbed in a certain price change level (Amihud (2002)).

Table 2 presents LOBL we used in this study. Cost of Round Trip Model (CRT(D)) is introduced to capture transactions of all sizes in limit order book (Irvine et al. (2000)). Best bid price is denoted by P_{-0} and best ask price is denoted by P_0. At the same time, lower levels of prices are denoted by the same rule, by which lower bid prices are denoted by $\{P_{-1} > P_{-2} > P_{-3} > \cdots > P_{-n}\}$, and higher ask prices are denoted by $\{P_1 < P_2 < P_3 < \cdots < P_n\}$. Quantities corresponded to prices are denoted by $\{Q_{-n}, \ldots, Q_{-2}, Q_{-1}, Q_{-0}, Q_0, Q_1, Q_2, \ldots, Q_n\}$. Then the number of shares corresponded to dollar amount D should be decided. It represents the number of shares corresponding to a dollar amount D that can be traded at mid-point price which is the average price of best bid price and best ask price.

$$T(D) = 2D/(P_{-0} + P_0) \tag{1}$$

Two indicator functions I_{-k} and I_k are calculated to represent buy and sell orders corresponded to dollar amount D.

$$I_{-k} = \begin{cases} 1: T > \Sigma_{i=-0}^{-k} Q_i \\ \frac{T - \Sigma_{i=-0}^{-k+1} Q_i}{Q_{-k}} : T > \Sigma_{i=-0}^{-k+1} Q_i \text{ and } T < \Sigma_{i=-0}^{-k} Q_i \\ 0 : \text{otherwise} \end{cases} \tag{2}$$

$$I_{-k} = \begin{cases} 1: T > \Sigma_{i=0}^{k} Q_i \\ \frac{T - \Sigma_{i=0}^{k} Q_i}{Q_{-k}} : T > \Sigma_{i=0}^{k-1} Q_i \text{ and } T < \Sigma_{i=0}^{k} Q_i \\ 0 : \text{otherwise} \end{cases} \tag{3}$$

Therefore, CRT(D) measure is determined from conditions mentioned above as the per dollar trading cost of a roundtrip trade of dollar amount D, given by

$$CRT(D) = \frac{\Sigma_{k=0}^{\infty} I_k P_k Q_k - \Sigma_{k=0}^{\infty} I_{-k} P_{-k} Q_{-k}}{D} \tag{4}$$

Dispersion liquidity measure is a spread-related LOB liquidity measure which defines the concentration of limit orders around the mid-price (Kang et al. (2013)). The formulation is given by,

$$LD_i = \frac{1}{2}\left(\frac{\Sigma_{j=1}^{n} \omega_j^{Buy} Dst_j^{Buy}}{\Sigma_{j=1}^{n} \omega_j^{Buy}} + \frac{\Sigma_{j=1}^{n} \omega_j^{Sell} Dst_j^{Sell}}{\Sigma_{j=1}^{n} \omega_j^{Sell}}\right) \tag{5}$$

In this formulation, Dst_j is the price interval between the jth best bid or ask and the next best quote. Hence, $Dst_j^{Buy} = Bid_{j-1} - Bid_j$, and $Dst_j^{Sell} = Ask_j - Ask_{j-1}$. When $j = 1$, Dst_j is the price interval between best bid or ask price and the mid quote. Dst_j is weighted by the size of limit orders ω_j.

To evaluate net dispersion which indicates dispersion direction, Modified Dispersion liquidity measure is introduced in our earlier work (Shen et al. (2016)).

$$DiffLD_i = \frac{1}{2}\left(\frac{\Sigma_{j=1}^{n} \omega_j^{Buy} Dst_j^{Buy}}{\Sigma_{j=1}^{n} \omega_j^{Buy}} - \frac{\Sigma_{j=1}^{n} \omega_j^{Sell} Dst_j^{Sell}}{\Sigma_{j=1}^{n} \omega_j^{Sell}}\right) \tag{6}$$

It takes the average of the difference between the bid and ask aggregate dispersion over limit order book. This measure indicates the imbalance in the concentration of the bid and ask limit orders

Table 3 explains the meaning of each variable used in the liquidity measures listed in Table 1 and Table 2.

2.3 Liquidity Measures Calculations

We substitute the preprocessed data into formulations given in Table 1 and Table 2. Since all the variables for TAQL have been calculated in the dataset, we are able to calculate TAQL directly. For Cost of Round Trip Model, two different dollar amount D are selected. First dollar amount D1 is set to be the amount of dollar that corresponds to 1% daily trading volume, and second dollar amount D2 is set to be the amount of dollar that corresponds to 2% daily trading volume. For Dispersion Model and Modified Dispersion Model, we study the best 1, 2, 3, 5, and 10 quotes. These five levels of dispersion measures are selected based on bisection method such that it characterizes the most concentrated area.

Though we have selected suitable time windows and have deleted the inactive rows, the high frequency data still contains missing values when there is no trade or quote in the some time intervals. It is assumed that liquidity cannot have great change in a very short period, so we replace NA and Infinite values by previous liquidity measures values. In this study, the previous liquidity condition will be inherited to the next time window when the liquidity condition is unmeasurable. Trading quantity (Q) is the total trading volumes in the time window, but it represents only the volumes traded but not the quantity of equity that market can absorb without moving price. As this measure cannot be negative, when there is no trade happening in the time interval, the value of zero reflects zero liquidity but it does not make sense. For log spread (LogSabs) and log relative spread of log prices (LogSrellog), they are not able to deal with negative inputs, however, we have a considerable number of negative inputs in this study. Based on the analysis, these three measures are deleted from the candidate liquidity measures. After these operation, we have the complete and consistent dataset with all target liquidity measures results.

3 Empirical Results

In this section, the empirical results are illustrated. First, we explain the correlation analysis results. We focus on the correlation between TAQL and LOBL groups, and we study the relation between liquidity correlation and price movements. Second, we present the analysis of the tail events of liquidity measures.

3.1 Correlation Analysis

3.1.1 Correlation Matrix

To quantify the correlation between each two liquidity measures, we perform correlation analysis by estimating the Pearson correlation coefficient, which is a measure of the linear relationship between two variables reflecting both strength and direction of the relationship.

X and Y are two lists of variables, and elements in the lists are named as x and y. Then the data can be displayed in a scatter diagram in which each point represents a pair of (x, y). Usually the independent variable is placed on the horizontal axis, while the dependent variable is placed on the vertical axis. The correlation coefficient of sample date is,

$$r = \frac{Cov(x,y)}{\sqrt{s_x^2 * s_y^2}} \tag{7}$$

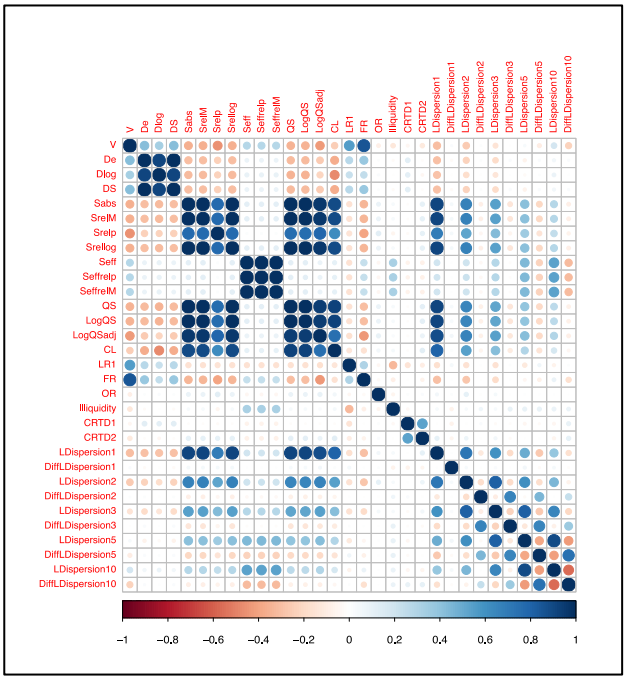
where $Cov\ (x, y)$ is the covariance of x and y,

$$Cov\ (x,y) = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{n-1} \tag{8}$$

s_x^2 and s_y^2 are variances of x and y,

$$s_x^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}\ and\ s_y^2 = \frac{\Sigma(Y-\bar{Y})^2}{n-1} \tag{9}$$

We can detect the common movements of the given liquidity set by employing correlation analysis. After that, we will have correlation matrix



which can present a clear view of relations among liquidity measures.

Fig. 1. Correlation mattrix heatmap of NEE on Day 1 (June 16, 2016)

The heatmap of Day 1 (June 16, 2016) is picked up to give a clear view of the correlation matrix heatmap. The heatmap in Figure 1 illustrates the correlations among 22 measures. The heatmap can be roughly divided into three parts. The upper left corner reflects correlations among TAQL, the upper right corner and the bottom left corner reflect correlations among TAQL and LOBL, and the bottom right corner reflects correlations among LOBL.

3.1.2 TAQL and LOBL Correlation

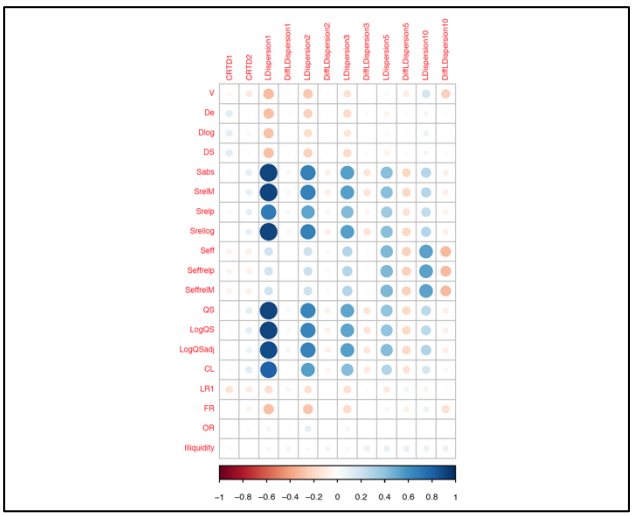


Fig. 2. TAQL and LOBL correlation matrix heatmaps of NEE on Day 1 (June 16, 2016)

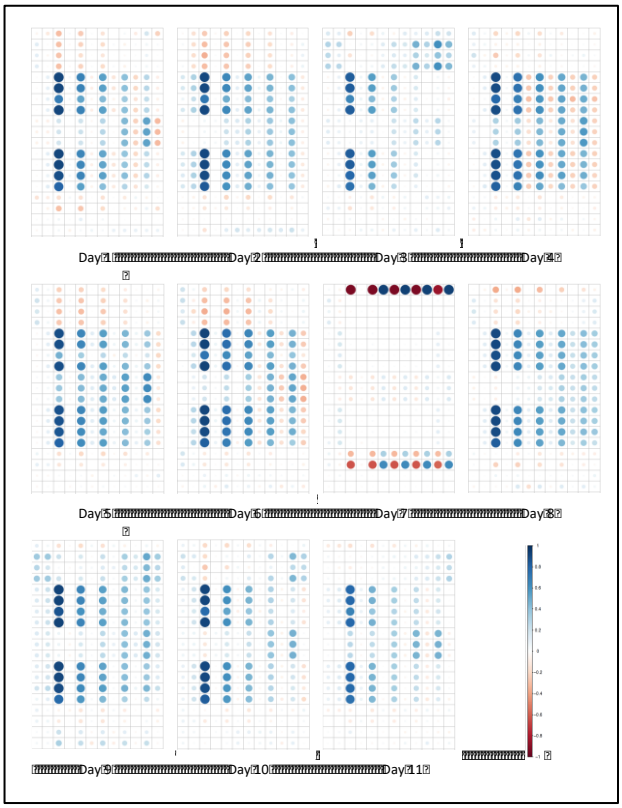


Fig. 3. TAQL and LOBL correlation matrix heatmaps of NEE

After constructing heatmaps of all 11 days, we notice that the right upper part has significant changes during the sample period. The symbol of NEE is taken as the example again, and the condition on Day 1 (June 16, 2016) is displayed in Figure 2. Then the blocks of correlation between TAQL and LOBL of all sample days are illustrated in Figure 3. It is noticed that Day 7 (June 24, 2016) is the special day after analyzing the 11 heatmaps, and the heatmap of Day 7 (June 24, 2016) is much different from the rest graphs.

3.1.3 Correlation Index

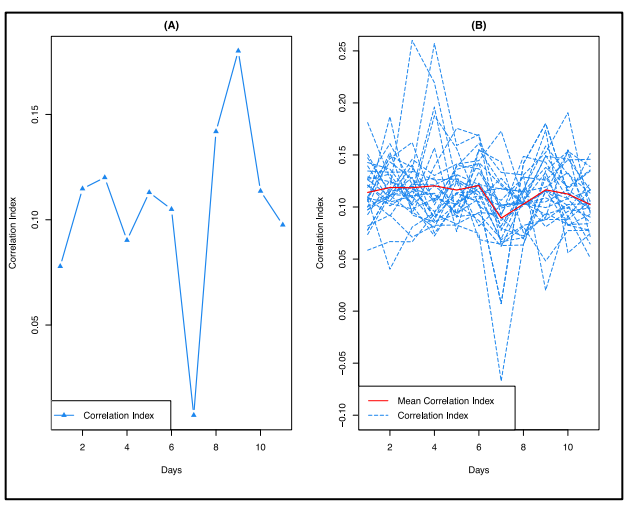


Fig. 4. NEE correlation index(A) and correlation indices with mean value (B)

However, heatmaps are subjective. We define a liquidity index to give a clearer view on the correlation conditions. In this study, we define the index as the average correlation of TAQL and LOBL correlation. The trend of symbol NEE is used as an example to make comparison with the heatmaps. Figure 4 is the trend of measures correlation indices, where (A) is the liquidity index of symbol NEE and (B) is the liquidity indices of all the sample with the mean value of them.

The heatmap representing Day 7 (June 24, 2016) indicates that the correlation drops, and we observe that the correlation index trend corresponds to this phenomenon as well. From Figure 4, it is clear that the correlation index (A) has a huge jump from Day 6 (June 23, 2016) to Day 7 (June 24, 2016). This phenomenon indicates that TAQL and LOBL are relatively highly correlated before and after the event day, but on the event day, the two liquidity groups become less correlated. When it comes to all the sample components, the average correlation index also has the same movements illustrated in graph (B).

To capture the large changes in the sample period, we are calculating the relative change in the correlation index. We set the threshold for large change and correlation jump as 17%. There are 14 components that have correlation jumps on Day 7 (June 24, 2016) accounting for 50%. And we also study the absolute correlation, which is defined as the average value of all absolute correlations. Absolute correlation focuses only on the strength. A criterion of 20% is set to be the signal of large absolute correlation changes. On the event day, large absolute correlation changes happened on 16 tickers, account for 57.14% of sample tickers. The results show that more than half of the sample has large correlation change on the event day indicating that Brexit has great influence on correlation between TAQL and LOBL. Table 4 gives the correlation change and absolute correlation change results. Because of the limit of page, only four days are shown in the table.

Table 4. Correlation index change and absolute correlation index change

	Correlation Index Change				Absolute Correlation Index Change			
DAY	22-Jun	23-Jun	24-Jun	27-Jun	22-Jun	23-Jun	24-Jun	27-Jun
NEE	-0.131	0.162	-0.488	0.764	0.251	-0.071	-0.932	18.802
DUK	0.153	0.203	-0.334	0.511	0.002	-0.099	-1.903	-1.956
SO	0.119	0.098	-0.019	0.268	0.279	0.041	-0.44	-0.092
D	-0.172	0.149	-0.011	0.177	-0.109	0.134	-0.017	0.402
EXC	-0.114	-0.013	0.279	-0.065	-0.295	0.484	-0.259	0.048
PCG	-0.036	-0.028	0.119	-0.123	0.084	0.024	0.196	-0.382
AEP	0.167	-0.256	0.289	-0.078	0.386	-0.474	-0.066	0.258
SRE	0.025	0.154	-0.188	-0.035	0.211	-0.154	-0.285	0.706
EIX	0.286	0.072	-0.254	0.077	0.545	-0.34	0.246	0.022
PPL	-0.052	0.098	0.228	-0.14	0.006	-0.098	0.031	-0.035
ED	-0.024	0.074	0.101	-0.046	0.19	0.009	-0.418	0.057
PEG	0.126	-0.027	0.07	0.096	0.085	0.32	-0.268	-0.229
XEL	0.162	-0.128	0.191	-0.122	0.083	-0.074	-0.035	-0.02
WEC	-0.063	0.235	-0.285	0.287	-0.052	0.185	-0.328	0.427
DTE	-0.128	0.054	-0.324	0.247	-0.381	0.065	-0.458	0.619
ES	0.118	0.019	-0.19	-0.005	0.261	-0.041	-0.626	0.001
AWK	0.104	-0.017	0.004	0.062	-0.513	1.025	-0.074	-0.474
FE	-0.131	0.042	0.092	-0.082	0.183	-0.111	-0.092	0.063
AEE	0.181	-0.116	-0.079	0.055	0.459	0.05	-0.38	0.199

ETR	0.223	-0.159	-0.075	-0.038	0.358	0.161	-0.172	-0.018
CMS	-0.097	-0.023	0.31	-0.055	0.384	0.378	-0.154	-0.155
CNP	-0.004	0.132	-0.07	-0.082	-0.159	0.651	-0.211	-0.159
SCG	-0.241	0.027	-0.059	0.111	-0.538	0.029	-0.255	0.645
PNW	-0.17	-0.064	0.013	0.128	-0.437	-0.026	-0.355	0.506
LNT	-0.011	-0.202	0.33	-0.305	-0.168	-0.353	-0.137	0.164
AES	0.047	-0.073	0.132	0.051	-0.165	0.145	0.184	-0.196
NI	0.068	-0.034	0.178	-0.083	0.617	-0.025	-0.259	0.021
NRG	-0.138	0.119	-0.083	0.137	-0.148	0.153	-0.101	0.21

3.1.4 TAQL and LOBL Correlation

The Brexit referendum started at 2 am EST on June 23 and ended at 2 am EST the next day. This indicates that the entire day of June 23 is covered by the referendum and there is a time gap between the result release and trading hours start. Since the voting results were released region by region, June 23 is influenced by continuous information, and before trading hours started on June 24, information was gathered in 7.5 hours. Table 5 illustrates daily last price change percentages. We set 1% as the criteria of price jump. The number of underlying which occur price jump is 21, accounting for 75%, and when we consider top 2 largest last price changes in the sample time period for each symbol (if there is no price change that hits the threshold), the total number of constituents reaching this level is 26, accounting for 92.86%. The results show that most components have price jumps on the event day.

Table 5. Last price change

DAY	20-Jun	21-Jun	22-Jun	23-Jun	24-Jun	27-Jun	28-Jun
NEE	-0.0121	0.0017	-0.0002	-0.0027	0.0209	0.0112	-0.0054
DUK	-0.0157	0.0004	0.002	-0.0027	0.0127	0.0076	0.0015
SO	-0.0118	0.001	0.0016	-0.0024	0.0119	0.0115	0.0052
D	-0.0085	0.0001	-0.0005	-0.0027	0.0071	0.0037	0.0067
EXC	-0.0058	0.0012	-0.0049	-0.0011	0.0093	-0.0064	-0.0054
PCG	-0.0128	0.0006	-0.0042	-0.0035	0.0156	0.0035	-0.0067
AEP	-0.0108	0.0004	-0.0005	-0.0041	0.014	0.0018	0
SRE	-0.0063	0.0066	-0.0004	-0.0019	0.0047	-0.0018	-0.0075
EIX	-0.0093	0.0022	-0.0018	-0.0046	0.0125	0.0089	-0.018
PPL	-0.0015	-0.0023	-0.0053	-0.0025	0.0097	-0.0027	-0.0101
ED	-0.019	-0.0025	0.0019	-0.0047	0.0149	0.0127	-0.0049
PEG	-0.0099	0.0005	-0.0007	-0.0038	0.0095	0.0029	-0.0076
XEL	-0.014	0.0011	0.0002	-0.0019	0.023	0.003	-0.0066
WEC	-0.0153	0.0006	0.0006	-0.0054	0.0167	0.0037	-0.0115
DTE	-0.0122	0.002	-0.0009	-0.0053	0.0185	0.0019	-0.0086
ES	-0.0069	0.0028	0.0011	-0.0046	0.0187	0.0051	-0.0144
AWK	-0.0182	0.0052	-0.0004	0.0014	0.0144	0.0087	-0.017
FE	-0.0077	-0.0051	0.0003	-0.0048	0.0097	-0.0042	0.0039
AEE	-0.009	0.0027	-0.001	0.0002	0.0131	0.0038	-0.0145
ETR	-0.0099	0.0008	0.0027	-0.0031	0.0114	-0.0025	-0.0081
CMS	-0.014	-0.0016	0.0025	-0.0046	0.0248	0.0024	-0.0104
CNP	-0.003	0.003	0.0017	-0.0021	0.0128	-0.0056	-0.0047
SCG	-0.0098	0.0035	0.0007	-0.0039	0.0123	0.0055	-0.0104
PNW	-0.01	0.002	-0.0006	0.0049	0.0105	0.0024	-0.0127
LNT	-0.0105	0.0018	-0.0021	-0.0023	0.0227	0.001	-0.0141
AES	0.0072	0.01	-0.0025	-0.0025	0.0135	-0.0281	-0.011
NI	-0.0076	0.0055	-0.0004	-0.0052	0.0111	0.0071	-0.0016
NRG	0.0063	-0.0028	-0.0241	0.0062	0.0057	-0.021	0.013

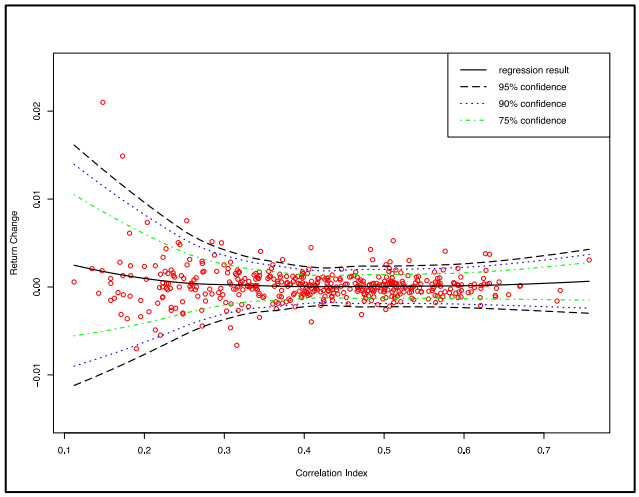


Fig. 5. Correlation index and return change of NEE

Figure 5 illustrates the relation between correlation index and return change on NEE. The black solid line represents the regression result, while the black dashed line, blue dotted line, and green dashed line represent 95%, 90%, and 75% confidence intervals respectively. It is obvious that all levels of confidence intervals have the tendency to converge with the increase of correlation index. This phenomenon indicates that when liquidity correlation index is low, it has high probability to have large price change. This corresponds to the previous discovery in Table 5 that prices of most tickers in the sample have jumps and at that time when liquidity correlation index drops.

3.2 Tail Events Study

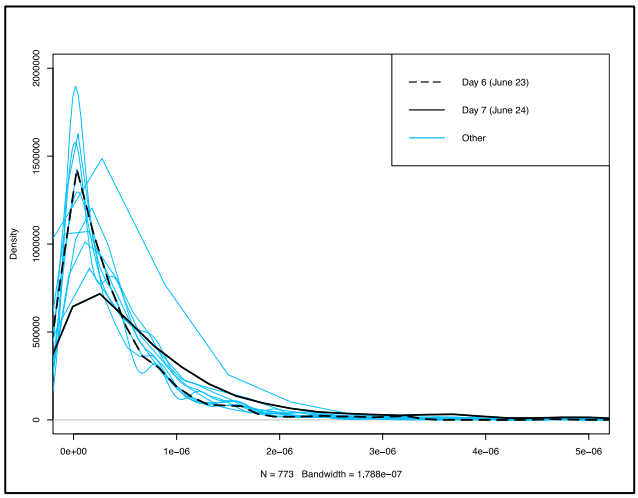


Fig. 6. Illiquidity (Amihud) distribution on NEE with Day 6 (June 23, 2016) and Day 7 (June 24, 2016) highlighted

We follow the methodology used in Luo et al. (2013) to perform analysis on liquidity measures distributions. Given a sequence of independent distributed random variables, we use Gaussian Kernel Density Estimation (KDE) for the probability density function (Parzen (1962)). Illiquidity measure is widely used to reflect market liquidity; therefore, Illiquidity is selected as the example to illustrate methods and results. Symbol NEE is selected again as the example for test. In Figure 6, it is observed from the graph that these 11 lines have similar shapes. The lines representing Day 6 (June 23, 2016) and Day 7 (June 24, 2016) are marked by black dashed line and black solid line respectively.

It is important to remind that Day 6 (June 23, 2016) is the voting day for Brexit and Day 7 (June 24, 2016) is the event day. The black dashed line has a sharp peak, while the black solid line has a small peak. By the definition, kurtosis is a descriptor of the shape of a probability distribution, and higher kurtosis is the result of infrequent extreme deviations (or outliers), as opposed to frequent modestly sized deviations. Because of this, the sharp peak of the black dashed lines is caused by infrequent low liquidity condition. In Day 6 (June 23, 2016), the liquidity conditions are in a stable range in which few extreme liquidity conditions happened causing the sharp peak in the graph of the black dashed line. However, the black solid line has the opposite trend. It has the smallest peak among all 11 lines. It tells that in Day 7 (June 24, 2016), low liquidity conditions occurred frequently.

All the tickers considered are analyzed and additional Illiquidity distributions are presented in Figure 7. The black dashed lines have sharp peak caused by infrequent extreme values and black solid lines have fat tails reflecting frequent extreme illiquid conditions. This phenomenon is observed in all the other symbols.

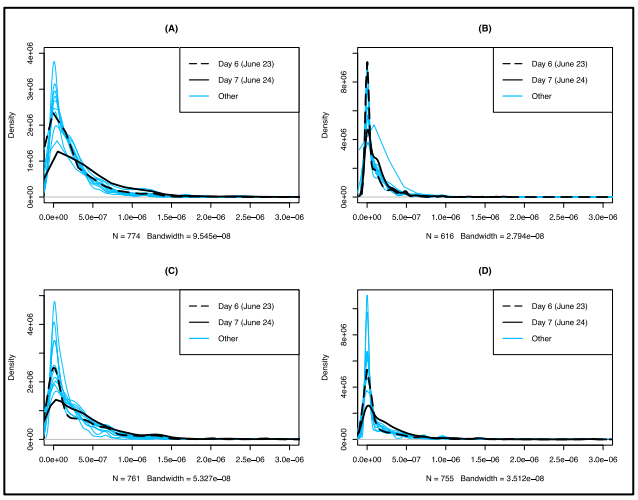


Fig. 7. Illiquidity (Amihud) distribution on DUK(A), SO(B), D(C) and EXC(D) with Day 6 (June 23, 2016) and Day 7 (June 24, 2016) highlighted

Also, different liquidity measures are tested to check the extent of this phenomenon. Figure 8 shows different measures distributions on the symbol of NEE. Lines representing Day 6 (June 23, 2016) and Day 7 (June 24, 2016) are marked black dashed and black solid respectively. From the graphs, we can see that they all have the same behavior as mentioned before that measures on Day 7 (June 24, 2016) have fatter tails.

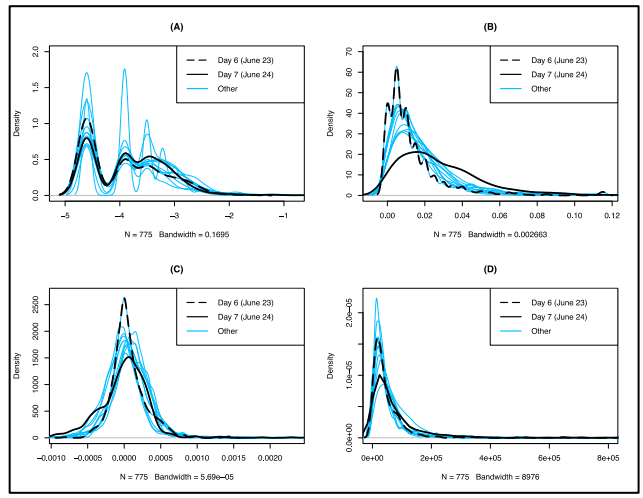


Fig. 8. LogSabs(A), Seff(B), Srellog(C), and V(D) distributions on NEE with Day 6 (June 23, 2016) and Day 7 (June 24, 2016) highlighted

Besides graphs, statistics are presented for a clear view of the distributions. We study quantiles to give a quantitative analysis. In statistics and theory of probability, quantiles are the cutpoints dividing the range of a probability distribution into contiguous intervals with equal probabilities or dividing the sample into equal-sized interval. The cutpoints locations can describe the difference of each liquidity measures distributions. The movements of the same level quantile cutpoints of one liquidity measure indicate the thickness of distribution tails.

Table 6. Percentage of quantiles' movement to illiquidity (Amihud) direction on Day 7(June 24, 2016) relative to Day 6 (June 23, 2016)

Quantiles	75%	90%	95%	99%	99.50%	99.90%
Sabs	21.43%	25.00%	35.71%	46.43%	42.86%	46.43%
LogSabs	25.00%	28.57%	28.57%	42.86%	42.86%	46.43%
SrelM	21.43%	39.29%	53.57%	60.71%	60.71%	60.71%
Srelp	17.86%	35.71%	53.57%	57.14%	53.57%	57.14%
Srellog	21.43%	39.29%	53.57%	60.71%	60.71%	60.71%
LogSrellog	67.86%	53.57%	64.29%	64.29%	64.29%	60.71%
Seff	57.14%	82.14%	78.57%	75.00%	67.86%	64.29%
Seffrelp	85.71%	85.71%	89.29%	78.57%	71.43%	67.86%
SeffrelM	85.71%	85.71%	89.29%	78.57%	71.43%	64.29%
QS	39.29%	32.14%	57.14%	57.14%	64.29%	64.29%
LogQS	39.29%	28.57%	50.00%	60.71%	57.14%	60.71%
LogQSadj	46.43%	35.71%	50.00%	57.14%	53.57%	53.57%
CL	39.29%	39.29%	53.57%	57.14%	53.57%	67.86%
OR	32.14%	39.29%	42.86%	60.71%	53.57%	35.71%
Amihud	60.71%	75.00%	71.43%	60.71%	67.86%	71.43%
Illiquidity	64.29%	75.00%	67.86%	60.71%	67.86%	71.43%
CRTD1	17.86%	10.71%	21.43%	50.00%	50.00%	42.86%
CRTD2	35.71%	21.43%	25.00%	28.57%	42.86%	53.57%
V	67.86%	60.71%	53.57%	57.14%	57.14%	64.29%

De	53.57%	53.57%	46.43%	53.57%	46.43%	46.43%
Dlog	57.14%	53.57%	46.43%	53.57%	46.43%	46.43%
DS	71.43%	75.00%	71.43%	78.57%	71.43%	67.86%
LR1	71.43%	75.00%	64.29%	60.71%	60.71%	71.43%
FR	57.14%	50.00%	53.57%	53.57%	53.57%	64.29%
DiffLD1	14.29%	0.00%	0.00%	17.86%	32.14%	39.29%
DiffLD2	78.57%	85.71%	82.14%	67.86%	60.71%	57.14%
DiffLD3	64.29%	67.86%	67.86%	67.86%	78.57%	75.00%
DiffLD5	60.71%	64.29%	67.86%	67.86%	67.86%	57.14%
DiffLD10	50.00%	50.00%	53.57%	50.00%	50.00%	53.57%
LD1	28.57%	32.14%	39.29%	42.86%	42.86%	46.43%
LD2	57.14%	67.86%	78.57%	71.43%	75.00%	67.86%
LD3	78.57%	75.00%	75.00%	64.29%	71.43%	71.43%
LD5	71.43%	71.43%	67.86%	67.86%	75.00%	64.29%
LD10	71.43%	60.71%	64.29%	75.00%	67.86%	67.86%
Quantiles	75%	90%	95%	99%	99.50%	99.90%
Sabs	21.43%	25.00%	35.71%	46.43%	42.86%	46.43%
LogSabs	25.00%	28.57%	28.57%	42.86%	42.86%	46.43%
SrelM	21.43%	39.29%	53.57%	60.71%	60.71%	60.71%
Srelp	17.86%	35.71%	53.57%	57.14%	53.57%	57.14%
Srellog	21.43%	39.29%	53.57%	60.71%	60.71%	60.71%
LogSrellog	67.86%	53.57%	64.29%	64.29%	64.29%	60.71%
Seff	57.14%	82.14%	78.57%	75.00%	67.86%	64.29%
Seffrelp	85.71%	85.71%	89.29%	78.57%	71.43%	67.86%
SeffrelM	85.71%	85.71%	89.29%	78.57%	71.43%	64.29%
QS	39.29%	32.14%	57.14%	57.14%	64.29%	64.29%
LogQS	39.29%	28.57%	50.00%	60.71%	57.14%	60.71%
LogQSadj	46.43%	35.71%	50.00%	57.14%	53.57%	53.57%
CL	39.29%	39.29%	53.57%	57.14%	53.57%	67.86%
OR	32.14%	39.29%	42.86%	60.71%	53.57%	35.71%
Amihud	60.71%	75.00%	71.43%	60.71%	67.86%	71.43%
Illiquidity	64.29%	75.00%	67.86%	60.71%	67.86%	71.43%
CRTD1	17.86%	10.71%	21.43%	50.00%	50.00%	42.86%
CRTD2	35.71%	21.43%	25.00%	28.57%	42.86%	53.57%

In this paper, 75%, 90%, 95%, 99%, 99.5%, and 99.9% quantiles are analyzed to check if they have movements towards the illiquidity direction on the event day indicating fat tails on the illiquid part. The percentage values in the Table 6 indicates the percentage of sample ETF components' quantiles have the movements. According to the results of all the liquidity measures we analyzed, liquidity measures generally have the movements towards illiquid direction on the event day. To be more specific, 99 % and 99.5 % quantiles have better performance to describe the movements. We can observe that with the increase of quantile ordinal, the percentage values increase, however, the percentage values under 99.9% quantile do not continue this trend. This phenomenon indicates that illiquid condition happens frequently on the event day causing fatter tails, but extremely low liquidity condition does not occur frequently.

From line chart analysis and quantile analysis, we find that liquidity distributions are usually platykurtic on the event day. Liquidity exhibits a low-peak and fat-tail shape at event day.

4 Conclusion

In this study, we investigate a large number of liquidity measures applied to both Trade & Quote (TAQ) data and Limit Order Book (LOB) data. Specifically, we investigate the correlation between TAQL and LOBL. The main findings indicate that the correlation structure between these two liquidity groups change significantly on market event days. In general, the two groups exhibit a stable correlation structure, but on the event day, most of the correlations drop to insignificant values, while some inactive liquidity pairs show large positive and negative correlations.

The tail analysis on the distributions of liquidity measures shows an increasing occurrence in the tail events across all ETF components studied. The investigation of the TAQL and LOBL quantiles shows that the distributions of the liquidity measures have small peaks and fatter tails on significant market event days.

Both methodologies implemented in this paper provide tools for identification of increasing probabilities of large price movements of the underlying securities and illiquid market conditions.

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