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Parameters Affecting the Efficacy and Relative Location of a Distribution Center in a Supply Chain with Fixed Replenishment Intervals

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Abstract

This study addresses the efficacy and placement of a distribution center in a supply chain consisting of a single supplier and multiple retailers. The DC and retailers follow a fixed interval, order-up-to replenishment policy for a single product. The retailers experience independent, identically-distributed demand and unsatisfied demand at the DC and retailers is backlogged. Lead times for the DC and retailers are constant. With the goal in mind of minimizing total system safety stock, six fundamental questions are addressed. For example, how many retailers must be present in the network for a centralized system to be superior to a decentralized system? For another, should the DC be closer to the supplier or the retailers? One or more equations are presented to help answer each question. This study adds to the research on supply chain design and inventory pooling by showing the impact of system parameters on safety stock requirements.

Keywords: multi-echelon inventory system, supply chain design, inventory risk pooling, centralized safety stock

1 Introduction

Two types of inventory system are contrasted in this paper: decentralized (one-echelon) and centralized (two-echelon). In the decentralized system, retailers are directly replenished by the supplier/source of the product. In the centralized system, retailers get the product from an intermediary (distribution center) that in turn is replenished by the supplier.

The argument – not always true – for the use of centralized inventory at a distribution center (DC) is that it requires lower overall safety stock. The logic cited is that when each separate retailer must maintain its own safety stock, a higher level is required than would be if a portion of safety stock is held at a centralized facility. For sure, the centralized system allows for delayed allocation of inventory from markets with lower demand to those with higher demand, but in some cases this arrangement requires more, not less, safety stock. This paper examines the factors affecting the validity of the argument for centralization.

An important consideration in contemplating inventory centralization is whether the physical product is suitable for a multi-echelon system. Products having costly storage or handling requirements might not be good candidates for pooling. The same holds true for products that can be easily damaged due to repeat handling. Unsuitable for centralization might also be products with very short shelf life if centralization results in their being in the supply chain significantly longer. A centralized system is most effective in service or manufacturing operations involving materials that have relatively stable and predictable demand. In this study we assume the product involved is suitable for inventory centralization.

2 Literature Review

Research on the general concept of inventory centralization (also referred to as risk pooling, aggregation and consolidation) goes back at least three decades. Zinn et al. (1989) shows the mathematical reasoning, what they call the "square root law", behind the potential reduction in necessary safety stock resulting from pooling. Numerous authors address the effects of pooling on service levels and costs, including: Abdul-Jalbar et al. (2003), Benjaafar et al. (2005), Berman et al. (2011), Li and Zhang (2012), Tagaras (1989), and Wanke and Saliby (2009).

Looking beyond the general pros and cons of centralization, research has been published that runs the gamut of related topics. For example, different network configurations have been studied including: three-plus echelons (Kang and Kim, 2012), multiple suppliers (Ganeshan, 1999; Park et al., 2010), retailers as distributors (Shen et al., 2003; Tagaras, 1999), and facility capacity (Kumar and Tiwari, 2013).

Researchers have looked at various ways of responding to inventory shortages caused by, for example, supply disruptions (Schmitt et al., 2015;

Zhang et al., 2016). Responses to shortages that have been studied include: inventory allotment (Edirisinghe and Atkins, 2017; Graves, 1996; Jackson, 1988; Jackson and Muckstadt, 1989; Marklund and Rosling, 2012; McGavin et al., 1993), transshipment (Evers, 1996; Tagaras, 1999; Zhang, 2005), redistribution (Jonsson and Silver, 1987); and product substitution (Yang and Schrage, 2009).

Different demand distributions (Yang and Schrage, 2009; Dai et al., 2017) and demand variability (Gerchak and He, 2003) have been studied. Cost consideration in some papers has gone beyond just inventory holding cost. Das and Tyagi (1997) include transportation costs, Shang et al. (2015) consider fixed order costs, and Romeijn et al. (2007) factor in fixed DC operating costs.

The bulk of the research on multi-echelon systems assumes a multi-period operation with inventory carried over from period to period. There are, however, a number of researchers (Cai and Du, 2009; Chang and Lin, 1991; Cherikh, 2000; Eppen, 1979; Stulman, 1987) who examine the effects of centralization in the context of the single-period (a.k.a. newsboy or newsvendor) problem. While these works are insightful, the conclusions drawn have limited applicability to the multi-period problem studied here.

This research is intended to bring clarity to some rudimentary principles underlying the centralization of inventory.

3 Research Model

3.1 Assumptions and Input Parameters

The supply chain model studied here assumes:

- a single product;
- a single source with unlimited supply;
- a time unit of one week;
- a weekly order interval for the DC and retailers;
- order-up-to replenishment, with no minimum order quantity, for the DC and retailers;
- identical, independent, normally-distributed demand at the retailers;
- a desired service level at the DC and retailers equaling the probability of meeting all demand as it occurs;
- backlogging unsatisfied demand at the DC and retailers;
- no transshipments between retailers; and
- known, constant lead times for the DC and retailers (equal for the retailers).

Input parameters include the number of retailers, retailer average weekly demand and standard deviation of demand, supplier-to-retailer lead time for a decentralized system, supplier-to-DC and DC-to-retailer lead times for a centralized system, and desired service level.

3.2 Mathematical Framework

The notation used herein includes:

S_D total safety stock required in the decentralized supply chain
S_C total safety stock required in the centralized supply chain
z z-score based on the desired service level
σ_d weekly demand standard deviation
N number of retailers
$L_{S\text{-}R}$ lead time from supplier to retailer in the decentralized supply chain
$L_{S\text{-}D}$ lead time from supplier to distribution center in the centralized supply chain
$L_{D\text{-}R}$ lead time from distribution center to retailer in the centralized supply chain

Assuming an order interval of one week at both the DC and the retailers, the periods of demand uncertainty (PODU), for the purpose of calculating order quantities, are denoted:

$L_{S\text{-}R} + 1$ period of demand uncertainty for a retailer in the decentralized supply chain
$L_{S\text{-}D} + 1$ period of demand uncertainty for the DC in the centralized supply chain
$L_{D\text{-}R} + 1$ period of demand uncertainty for a retailer in the centralized supply chain

Equations (1) and (2) determine the required safety stock, S_D and S_C, for a decentralized system and a centralized system, respectively.

$$S_D = z\sigma_d N\sqrt{L_{S-R}+1} \tag{1}$$

$$S_C = z\sigma_d\left(\sqrt{N}\sqrt{L_{S-D}+1} + N\sqrt{L_{D-R}+1}\right) \tag{2}$$

For the sake of contrasting safety stock levels in the analyses that follow, we use in (1) and (2) a safety factor, z, set to 1.96, representing a desired 97.5% service level at both the DC and retailers. The time unit used for replenishment lead times and order intervals is a week. Also, the weekly demand standard deviation, σ_d, is set to 10.

4 Research Questions Addressed

4.1 Question 1

How many retailers are necessary in the supply chain for a centralized system to be superior to a decentralized system in minimizing required system safety stock?

Using (3), we can compute the equilibrium number of retailers, denoted N_E, which equalizes S_D and S_C.

$$N_E = \frac{L_{S-D}+1}{(L_{S-R}+1) - 2\sqrt{L_{S-R}+1}\sqrt{L_{D-R}+1} + (L_{D-R}+1)} \tag{3}$$

For example, given $L_{S\text{-}R} = 5$, $L_{S\text{-}D} = 4$, and $L_{D\text{-}R} = 1$:

$$N_E = \frac{4+1}{(5+1) - 2\sqrt{5+1}\sqrt{1+1} + (1+1)} = 5/1.0718 = 4.665$$

In this example, a decentralized system and centralized system are equal in required safety stock when there are 4.665 retailers. As evidenced in Table 1, with four or fewer retailers a decentralized system is superior, while with five or more retailers a centralized system is superior.

Hence, the number of retailers necessary for a centralized system to be superior depends on the three lead times: $L_{S\text{-}R}$, $L_{S\text{-}D}$, and $L_{D\text{-}R}$. Furthermore, $L_{S\text{-}R}$ does not have to equal $L_{S\text{-}D}$ plus $L_{D\text{-}R}$.

Table 1. Number of retailers and safety stock required (for $L_{S\text{-}R} = 5$, $L_{S\text{-}D} = 4$, $L_{D\text{-}R} = 1$).

	Safety Stock Units	
Number of Retailers, N	Retailers (Decentralized)	DC & Retailers (Centralized)
3	144.03	159.07
4	192.04	198.53
4.665	223.97	223.97
5	240.05	236.59
6	288.06	273.67

4.2 Question 2

For a centralized system to be superior to a decentralized system in minimizing system safety stock, must the DC be closer (in time) to the supplier or the retailers?

In this study we speak of facility "closeness" in a temporal, rather than spatial, sense. That is, supplier-to-DC and DC-to-retailer closeness is measured in time rather than distance. Hence, the PODU (period of demand uncertainty) for the DC and the retailers, ($L_{S\text{-}D}$ + 1) and ($L_{D\text{-}R}$ + 1) respectively, are measures of the relative closeness of the DC to the supplier and the DC to the retailers.

Using (4), we can compute F, the factor by which $L_{S\text{-}D}$ + 1 is multiplied to get $L_{D\text{-}R}$ + 1.

$$F = \frac{L_{S-R}+1-2\left({}^{1}\!/_{\sqrt{N}}\right)\left(\sqrt{L_{S-D}+1}\right)\left(\sqrt{L_{S-R}+1}\right)+{}^{1}\!/_{N}\left(L_{S-D}+1\right)}{L_{S-D}+1} \tag{4}$$

A value for F less than 1.0 indicates the retailers' PODU should be less than the distribution center's PODU, meaning the DC should be closer to the retailers than the supplier. An F value greater than 1.0 means the DC should be closer to the supplier.

For example, with $N = 9$, $L_{S\text{-}R} = 7$, and $L_{S\text{-}D} = 4$:

$$F = \frac{7+1-2\left({}^{1}\!/_{\sqrt{9}}\right)\left(\sqrt{4+1}\right)\left(\sqrt{7+1}\right)+{}^{1}\!/_{9}\,(4+1)}{5} = .86784$$

In this example, the F value of .86784 means the PODU for the retailers, $L_{D\text{-}R}$ + 1, should be .86784 (about 87 percent) of the PODU for the distribution center, $L_{S\text{-}D}$ + 1, in order for the required safety stocks in the decentralized and centralized systems to be equal. Specifically, $L_{D\text{-}R} + 1 = F(L_{S\text{-}D} + 1) = .86784(4 + 1) = 4.3392$ weeks. Hence, $L_{D\text{-}R}$ +1 must be less than 4.3392 weeks for the centralized system to be superior. This is demonstrated in Table 2, with the point of comparison being the safety stock of 498.93 required in the decentralized system using (1).

Table 2. $L_{S\text{-}D}$ +1 equilibrium proportion of $L_{S\text{-}R}$ +1 (for $N = 9$, $L_{S\text{-}R} + 1 = 8$, $L_{S\text{-}D} + 1 = 5$)

Period of Uncertainty (Weeks)			Safety Stock Required (Units)		
DC	Retailers	Retailer Prop.	DC	Retailers	Total
(na)	8	(na)	(na)	498.93	498.93
5	6	1.20	131.48	432.09	563.57
5	5	1.00	131.48	394.44	525.92
5	4.3392	0.86784	131.48	367.45	498.94
5	4	0.80	131.48	352.80	484.28
5	3	0.60	131.48	305.53	437.01

If the PODUs for the DC and retailers ($L_{S\text{-}D}$ + 1 and $L_{D\text{-}R}$ + 1, respectively) in the centralized system should equal the PODU for the retailers in the decentralized system, $L_{S\text{-}R}$ + 1, a simpler equation can be used to determine the relative location of the DC. Using (5), we can compute P_E, the ($L_{S\text{-}D}$ + 1) equilibrium proportion of ($L_{S\text{-}R}$ + 1). Note, P_E depends solely on the number of retailers in the supply chain.

$$P_E = \frac{4}{N+{}^{1}\!/_{N}+2} \tag{5}$$

For example, with $N = 9$:

$$P_E = 4/(9 + (1/9) + 2) = 4/11.111 = .36$$

In this example, the PODU for the distribution center, $L_{S\text{-}D}$ + 1, should be at least .36 (36 percent) of the PODU for the retailers, $L_{S\text{-}R}$ + 1, in a decentralized system. If, say, $L_{S\text{-}R} + 1 = 10$ weeks:

$$L_{S\text{-}D} +1 = P_E(L_{S\text{-}R} + 1) = .36(10) = 3.6 \text{ weeks}$$

$$L_{D\text{-}R} + 1 = (1 - P_E)(L_{S\text{-}R} + 1) = .64(10) = 6.4 \text{ weeks}$$

Hence, the PODU for the DC must be more than 3.6 weeks, and the PODU for the retailers must be less than 6.4 weeks, for the centralized system to be superior to the decentralized system. This is demonstrated in Table 3, with the point of comparison being the safety stock of 557.83 required in the decentralized system using (1).

Table 3. $L_{S\text{-}D}$ +1 equilibrium proportion of $L_{S\text{-}R}$ +1 (for $N = 9$, $L_{S\text{-}R} +1 = 10$)

$L_{S\text{-}R}$ + 1	Safety Stock Units at Retailers (Decentralized System)	Safety Stock Units at DC & Retailers (Centralized System)
14	220.01	234.21
15	227.73	234.21
15.87	234.21	234.21
16	235.20	234.21
17	242.44	234.21

Shown in Fig. 1 are the ($L_{S\text{-}D}$ + 1)/($L_{S\text{-}R}$ + 1) proportions for various retailer counts, N, that equalize the decentralized and centralized safety stock requirements. For a centralized system with $N = 6$, the DC being located about midway between the supplier and the retailers equalizes S_D and S_C. As N decreases from six the DC must be increasingly closer to the retailers. As N increases from six the DC must be increasingly closer to the supplier.

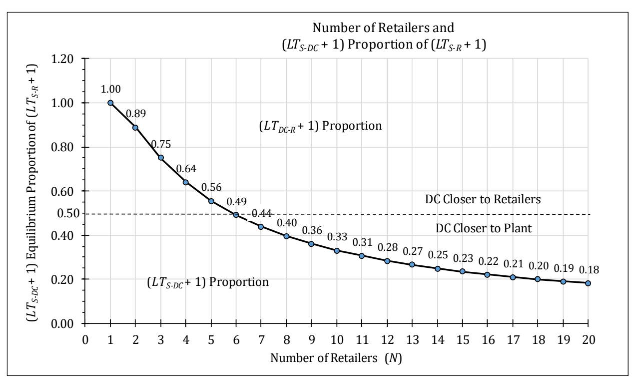


Fig. 1. ($L_{S\text{-}D}$ + 1)/($L_{S\text{-}R}$ + 1) proportions for S_D and S_C equilibrium

4.3 Question 3

Can a centralized system be superior, in minimizing system safety stock, when its combined period of demand uncertainty ($L_{S\text{-}D}$ + 1 and $L_{D\text{-}R}$ + 1) is greater than the decentralized system's period of demand uncertainty ($L_{S\text{-}R}$ + 1)? Conversely, can a decentralized system be superior when its PODU is greater than the centralized system's combined PODU?

The answers to both versions of the question are 'yes', under certain conditions. It depends on the relative closeness of the DC to the retailers and the number of retailers. Using (6), we can compute the equilibrium ratio, R_E, of $L_{S\text{-}R}$ + 1 to the combined $L_{S\text{-}D}$ + 1 and $L_{D\text{-}R}$ + 1.

$$R_E = \frac{(L_{S-D}+1)+2\sqrt{N(L_{S-D}+1)(L_{D-R}+1)}+N(L_{D-R}+1)}{N(L_{S-D}+L_{D-R}+2)} \qquad (6)$$

For example, given N = 3, $L_{S\text{-}D}$ = 3, and $L_{D\text{-}R}$ = 7:

$$R_E = \frac{(3+1)+2\sqrt{3(3+1)(7+1)}+3(7+1)}{3(3+7+2)} = \frac{46.4939}{36} = 1.3221$$

Hence, $L_{S-R}+1 = R_E(L_{S-D}+L_{D-R}+2) = 1.3221(3+7+2) = 15.8652$. In this example, safety stock required for a decentralized system with a PODU, $L_{S\text{-}R}$ + 1, of 15.8652 weeks equals the safety stock required for a centralized system with a combined PODU, $L_{S\text{-}D}$ + 1 plus $L_{D\text{-}R}$ + 1, of 12 weeks. A decentralized system is superior for $L_{S\text{-}R}$ + 1 $\leq$ 15, as evidenced in Table 4.

Table 4. $L_{S\text{-}R}$ +1 and required safety stock (for N = 3, $L_{S\text{-}D}$ + 1 = 4, $L_{D\text{-}R}$ + 1 = 8)

$L_{S\text{-}R}$ + 1	Safety Stock Units at Retailers (Decentralized System)	Safety Stock Units at DC & Retailers (Centralized System)
14	220.01	234.21
15	227.73	234.21
15.87	234.21	234.21
16	235.20	234.21
17	242.44	234.21

As another example, consider N = 15, $L_{S\text{-}D}$ = 7, and $L_{D\text{-}R}$ = 3:

$$R_E = \frac{(7+1)+2\sqrt{15(7+1)(3+1)}+15(3+1)}{15(7+3+2)} = \frac{111.8178}{180} = 0.62121.$$

Hence, $L_{S-R}+1 = R_E(L_{S-D}+L_{D-R}+2) = 0.62121(7+3+2) = 7.45452$. Safety stock required for a decentralized system with a PODU, $L_{S\text{-}R}$ + 1, of 7.45452 weeks equals the safety stock required for a centralized system with a combined PODU, $L_{S\text{-}D}$ + 1 plus $L_{D\text{-}R}$ + 1, of 12 weeks. A decentralized system is superior for $L_{S\text{-}R}$ + 1 $\leq$ 7, as evidenced by Table 5.

Table 5. $L_{S\text{-}R}$ +1 and required safety stock (for N = 15, $L_{S\text{-}D}$ + 1 = 8, $L_{D\text{-}R}$ + 1 = 4)

$L_{S\text{-}R}$ + 1	Safety Stock Units at Retailers (Decentralized System)	Safety Stock Units at DC & Retailers (Centralized System)
6	720.15	802.71
7	777.85	802.71
7.45	802.71	802.71
8	831.56	802.71
9	882.00	802.71

Fig. 2 shows the R_E values for various N and ($L_{S\text{-}D}$ + 1)/($L_{D\text{-}R}$ + 1) values including the R_E values found in the two examples immediately above. The first example's equilibrium ratio of 1.32 can be found using the N = 3 curve and a ($L_{S\text{-}D}$ + 1)/($L_{D\text{-}R}$ + 1) value of 0.50. The second example's equilibrium ratio of 0.62 can be found using the N = 15 curve and a ($L_{S\text{-}D}$ + 1)/($L_{D\text{-}R}$ + 1) value of 2.00.

At least two generalities can be drawn from Fig. 2. When there are very few retailers (e.g. N = 3) and the DC is closer to the supplier than the retailers, a decentralized system is superior (requires less safety stock) even when its PODU, $L_{S\text{-}R}$ + 1, is 30% greater than a centralized system's combined PODU, $L_{S\text{-}D}$ + 1 and $L_{D\text{-}R}$ + 1. Also, when there are six or more retailers and the DC is closer to the retailers, the decentralized system's PODU must be smaller than the centralized system's combined PODU for the decentralized system to be superior.

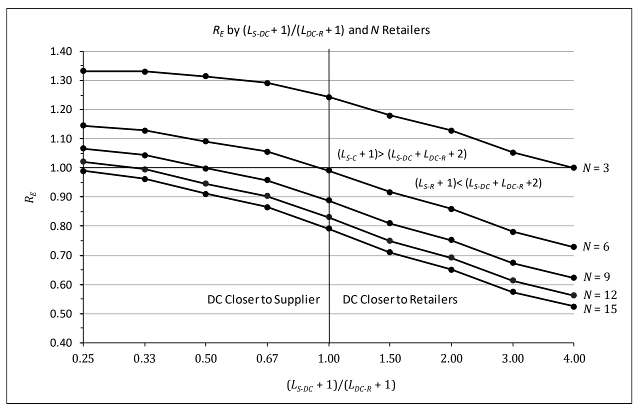


Fig. 2. R_E by ($L_{S\text{-}D}$ + 1)/($L_{D\text{-}R}$ + 1) and N retailers

4.4. Question 4

What is the percent change (increase or decrease) in system safety stock going from a decentralized system to a centralized system?

Using (7), we can compute C, the percent change in the supply chain's safety stock resulting from introducing a DC to the supply chain.

$$C = \left(\frac{{}^{1}/_{\sqrt{N}}\sqrt{L_{S-D}+1}+\sqrt{L_{D-R}+1}}{\sqrt{L_{S-R}+1}} - 1\right)100 \qquad (7)$$

For example, given N = 9, $L_{S\text{-}R}$ = 9, $L_{S\text{-}D}$ = 5, and $L_{D\text{-}R}$ = 3:

$$C = \left(\frac{1/\sqrt{9}\,\sqrt{5+1} + \sqrt{3+1}}{\sqrt{9+1}} - 1 \right) 100 = (.891 - 1)100 = -10.9\%$$

As shown in Table 6, the safety stock of 496.83 units required in the centralized system is 10.9% less than the 557.83 units required in the decentralized system.

Table 6. Decentralization-to-centralization change in safety stock (for $L_{S-R} = 9$, $L_{S-D} = 5$, $L_{D-R} = 3$)

	Safety Stock Units		Decentral.-
Number of	Retailers	DC & Retailers	to-Central.
Retailers, *N*	(Decentral.)	(Centralized)	Percent Change
3	185.94	200.76	8.0
6	371.88	352.80	-5.1
9	557.83	496.83	-10.9
12	743.77	636.71	-14.4
15	929.71	773.94	-16.8

Converting to a centralized system can result in an increase or a decrease in total safety stock, depending on the number of retailers, the closeness of the DC to the retailers, and the decentralized system's PODU relative to the centralized system's combined PODU.

Fig. 3a is based on the assumption the decentralized system's PODU equals the centralized system's combined PODU; that is, $L_{S-R} + 1$ equals $L_{S-D} + 1$ plus $L_{D-R} + 1$. As Fig. 3a indicates, as both the number of retailers and closeness of the DC to the retailers increase, the percent reduction in safety stock increases. Case in point: there is nearly a 1/3 reduction (specifically, -32.2%) in safety stock when there are 15 retailers and the DC is very close to the retailers, specifically, $(L_{S-D} + 1)/(L_{S-D} + L_{D-R} + 2) = .8$.

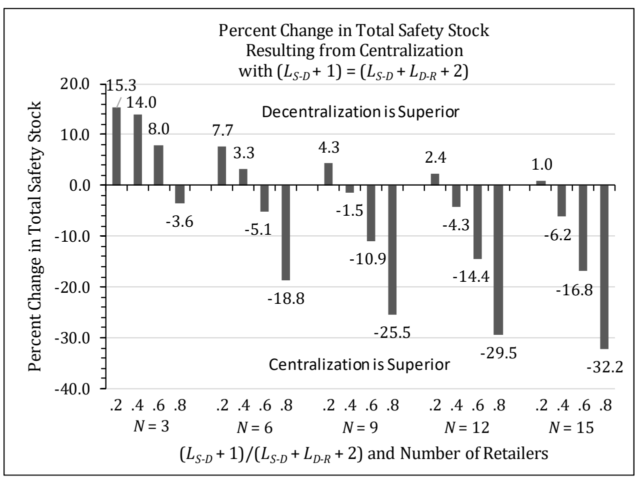
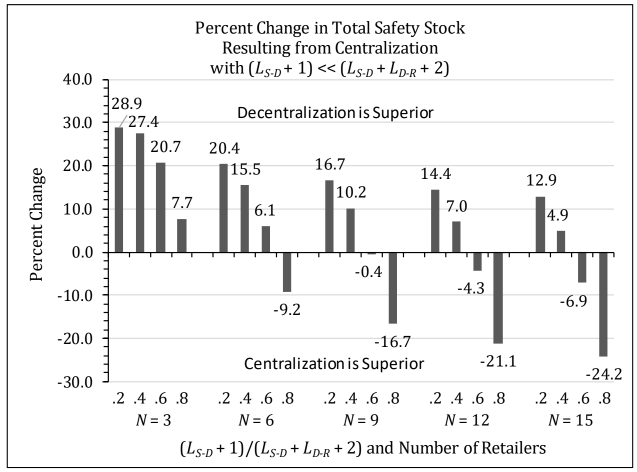


Fig. 3a. Percent change in safety stock from centralizing with $(L_{S-R} + 1) = (L_{S-D} + L_{D-R} + 2)$

It is evident from Fig. 3b that when the decentralized system's PODU, $L_{S-R} + 1$, is significantly greater than the centralized system's combined PODU, $L_{S-D} + L_{D-R} + 2$, there is a reduction in required safety stock regardless of where the DC falls on the supplier-to-retailer time line, except when the number of retailers is relatively small (e.g. $N = 3$).

Fig. 3b. Percent change in safety stock from centralizing with $(L_{S-R} + 1)$



>> $(L_{S-D} + L_{D-R} + 2)$

We see in Fig. 3c that when the decentralized system's PODU is significantly less than the centralized system's combined PODU the conditions under which there is a reduction in required safety stock are much more limited. In this case, the DC must be more than midway on the supplier-to-retailer time line (i.e. $(L_{S-D} + 1)/(L_{S-D} + L_{D-R} + 2) > .5$) and the number of retailers must be more than 3.

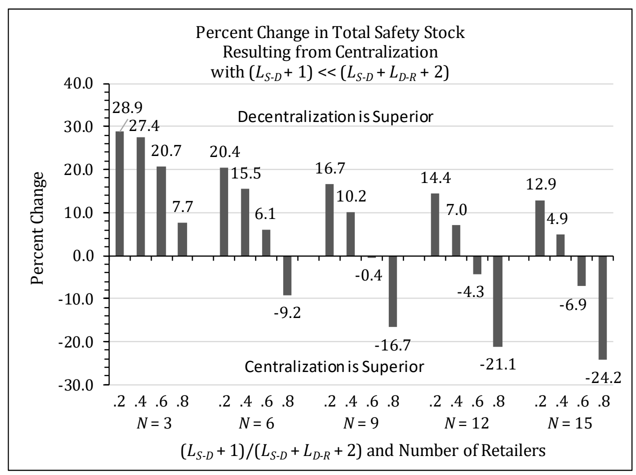


Fig. 3c. Percent change in safety stock from centralizing with $(L_{S-R} + 1)$ << $(L_{S-D} + L_{D-R} + 2)$

4.5 Question 5

Where is more safety stock held in a centralized system, at the DC or the retailers?

It depends on the DC's and retailers' PODUs, as well as the number of retailers. Using (8), we can compute the DC's proportion, P_{DC}, of total safety stock held.

$$P_{DC} = \frac{\sqrt{L_{S-D}+1}}{\sqrt{L_{S-D}+1} + \sqrt{N(L_{D-R}+1)}} \qquad (8)$$

For example, given $N = 2$, $L_{S\text{-}D} = 6$ and $L_{D\text{-}R} = 1$:

$$P_{DC} = \frac{\sqrt{6+1}}{\sqrt{6+1}+\sqrt{2(1+1)}} = .569$$

Hence, 56.9% of total safety stock will be held at the DC, and 43.1% at the retailers.

We can determine, for given $L_{S\text{-}D}$ and $L_{D\text{-}R}$ values, the number of retailers, N_B, required in a centralized system to equalize (balance) the safety stocks at the DC and the retailers by using (9).

$$N_B = \frac{L_{S-D}+1}{L_{D-R}+1} \tag{9}$$

For example, given $L_{S\text{-}D} = 6$ and $L_{D\text{-}R} = 1$:

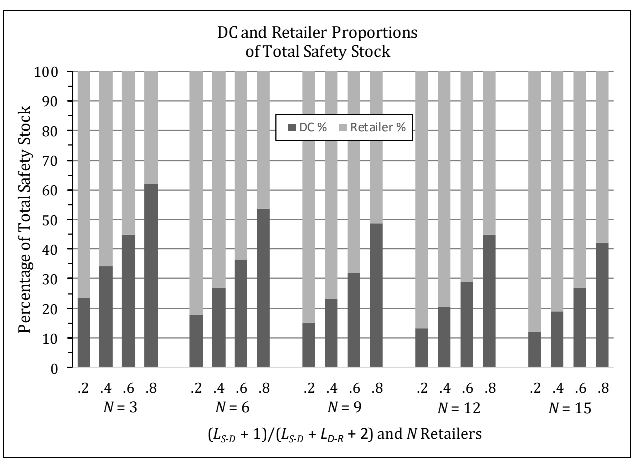
$$N_B = (6 + 1)/(1 + 1) = 3.5 \text{ retailers}$$

As evidenced in Table 7, with 2 retailers 56.9% of total safety stock is at the DC as calculated above. Furthermore, with 3 or fewer retailers in the chain the DC will be holding more than 50% of the chain's safety stock, but with 4 or more retailers the retailers will be holding the greater portion of total safety stock.

Table 7. Number of retailers and safety stock proportions (for $L_{S\text{-}D} = 6$ and $L_{D\text{-}R} = 1$).

	Safety Stock		Safety Stock Proportion	
Retailers, N	DC	Retailers	DC	Retailers
1	51.86	27.72	0.652	0.348
2	73.34	55.44	0.569	0.431
3	89.82	83.16	0.519	0.481
3.5	97.02	97.02	0.500	0.500
4	103.71	110.87	0.483	0.517
5	115.96	138.59	0.456	0.544
6	127.02	166.31	0.433	0.567

We see in Fig. 4 that as the DC becomes closer to the retailers (i.e. the greater $L_{S\text{-}D} + 1$ is relative to $L_{S\text{-}D} + L_{D\text{-}R} + 2$), the proportion of safety



stock held at the DC increases. It is also evident in Fig. 4 that as the number of retailers increases, an increasing proportion of total safety stock is held at the retailers.

Fig. 4. DC and retailer proportions of total safety stock

4.6 Question 6

Can using more than one DC, with no transshipping between them, be superior to using a single DC in minimizing system safety stock?

Let M denote the number of DCs in the supply chain. Using (10), we can compute M_E, the equilibrium number of DCs to no DC for S_D to equal S_C.

$$M_E = \frac{N((L_{S-R}+1)-2\sqrt{(L_{S-R}+1)(L_{D-R}+1)}+(L_{D-R}+1))}{L_{S-D}+1} \tag{10}$$

For example, given $N = 9$, $L_{S\text{-}R} = 10$, $L_{S\text{-}D} = 8$, and $L_{D\text{-}R} = 2$:

$$M_E = \frac{9((10+1)-2\sqrt{(10+1)(2+1)}+(2+1))}{(8+1)} = \frac{22.6}{9} = 2.511$$

Total safety stock required in a centralized system with M DCs is computed using (11):

$$S_{CM} = z\sigma_d\left(M\sqrt{N/M}\sqrt{L_{S-D}+1}+N\sqrt{L_{D-R}+1}\right) \text{ for } N/M \geq 1 \tag{11}$$

As shown in Table 8, total safety stock required in a centralized system with 2.511 DCs equals the amount required in a decentralized system with no DCs. In this example, using 1 or 2 DCs is superior to using no DC, but 1 DC is the best choice. Using 3 or more DCs is not beneficial.

Table 8. Number of DCs and safety stock required (for $N = 9$, $L_{S\text{-}R} = 10$, $L_{S\text{-}D} = 8$, $L_{D\text{-}R} = 2$)

Number of		Safety Stock Units	
DCs	DCs	Retailers	Total
0	0	585.05	585.05
1	176.40	305.53	481.93
2	249.47	305.53	555.00
2.511	279.53	305.53	585.06
3	305.53	305.53	611.07
4	352.80	305.53	658.33
5	394.44	305.53	699.98

A multiple-DC system, without transshipping, cannot be superior to a single DC, but can be superior to no DC. Total DC safety stock increases as M increases. The multiplying factor is $\sqrt{M}$. For $M = 2$, the factor is 1.414. We see in Table 8 that 2 DCs require 249.47 units of safety stock, which is 1.414 times the 176.40 units required by 1 DC. Similarly, when $M = 4$ the safety stock required (352.80) is exactly double the requirement for 1 DC.

5. Summary

In this study we addressed several fundamental questions that provide insight about both decentralization and centralization of inventory in a supply chain. We showed that whether centralization minimizes required safety stock for a supply chain with a DC having a given location relative to the supplier and retailers depends on how many retailers are in the chain. In addition, an equation was presented for determining the "break-even" number of retailers above which centralizing is beneficial.

While one might think that in a centralized system the DC should be closer (in time) to the retailers than the supplier, our analysis found that the optimal placement can go either way depending on the number of retailers involved. An equation was presented that finds how far along the supplier-to-retailer replenishment time line, at a minimum, the DC must be for centralization to be beneficial.

We learned that a centralized system with combined DC and retailer lead times *greater* than a decentralized system's retailer-only lead time can, in some cases, be superior in minimizing safety stock. Similarly, a decentralized system with retailer-only lead time *greater* than a centralized system's combined DC and retailer lead times can, in some cases, be superior. An equation was presented that determines the ratio of retailer-only lead time to the combined DC and retailer lead times that equalizes safety stock requirements.

Another topic addressed is the determination of the percent increase or decrease in total safety stock that results from implementing centralization. It was shown that direction and magnitude of change in total safety stock depends on the decentralized and centralized systems' periods of demand uncertainty and the number of retailers.

It was demonstrated that where more safety stock is held in a centralized system, at the DC or the retailers, depends on the system parameters (retailer count and PODUs). An equation was presented for calculating the safety stock proportion held at the DC for a given set of parameters. Another equation presented finds, for a given set of lead times, the number of retailers necessary for the DC and retailer proportions to be equal (at 0.5).

Finally, it was shown that using more than one DC, without transshipping, is never superior to using one DC in minimizing safety stock. However, multiple DCs *can* be superior to using none, and an equation was presented for determining the maximum number of DCs that would be superior to none.

This study lays the groundwork for a number of possible extensions stemming from the assumptions that are made here. The research questions addressed (as well as new questions) can be examined under different conditions, such as order intervals greater than one time unit, minimum order quantities, non-identical retailer demand distributions, other demand distributions than just normal, and transshipping between retailers.

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